

8.4 - Solving Equations with Variables on Both Sides:

$$ax = b + cx; ax + b = cx + d; a(bx + c) = d(ex + f)$$

Recall: The goal in solving equations is to isolate the variable.

To solve equations with variables on both sides, we need to first need to get the variable to one side only. We can then use the other algebraic techniques for solving that we have already learned.

- Steps:**
1. Use the distributive property to expand.
 2. Isolate the variable to one side of the equation.
 3. Solve for x.

Example 1:

a) $3(0.5x + 1.3) = 2(0.4x - 0.85)$

$$\begin{array}{r} 1.5x + 3.9 = 0.8x - 1.7 \\ -0.8x \quad -3.9 \quad -0.8x \quad -3.9 \\ \hline 0.7x = -5.6 \\ x = -8 \end{array}$$

check: $1.5(-8) + 3.9 = 0.8(-8) - 1.7$

$$\begin{array}{r} -12 + 3.9 = -6.4 - 1.7 \\ -8.1 = -8.1 \checkmark \end{array}$$

b) $6(x - 1.5) = 2x + 1.8$

$$\begin{array}{r} 6x - 9 = 2x + 1.8 \\ -2x \quad +9 \quad -2x \quad +9 \\ \hline 4x = 10.8 \\ \frac{4x}{4} = \frac{10.8}{4} \\ x = 2.7 \end{array}$$

check:

$$\begin{array}{r} 6(2.7) - 9 = 2(2.7) + 1.8 \\ 16.2 - 9 = 5.4 + 1.8 \\ 7.2 = 7.2 \checkmark \end{array}$$

Practice: Solve each equation

a) $5(2x + 1.2) = 4(x - 1.5)$

$$\begin{array}{r} 10x + 6 = 4x - 6 \\ -4x \quad -6 \quad -4x \quad -6 \\ \hline 6x = -12 \\ \frac{6x}{6} = \frac{-12}{6} \\ x = -2 \end{array}$$

check: $10(-2) + 6 = 4(-2) - 6$

$$\begin{array}{r} -20 + 6 = -8 - 6 \\ -14 = -14 \checkmark \end{array}$$

b) $1.3x + 64.2 = 2.7x + 12.82$

$$\begin{array}{r} -2.7x \quad -64.2 \quad -2.7x \quad -64.2 \\ \hline -1.4x = -51.38 \\ \frac{-1.4x}{-1.4} = \frac{-51.38}{-1.4} \\ x = 36.7 \end{array}$$

check:

$$\begin{array}{r} 1.3(36.7) + 64.2 = 2.7(36.7) + 12.82 \\ 47.71 + 64.2 = 99.09 + 12.82 \\ 111.91 = 111.91 \checkmark \end{array}$$

c) $1.2x - 17 = 8 + 0.7x$

$$\begin{array}{r} 0.5x = 25 \\ \frac{0.5x}{0.5} = \frac{25}{0.5} \\ x = 50 \end{array}$$

check: $1.2(50) - 17 = 8 + 0.7(50)$

$$\begin{array}{r} 60 - 17 = 8 + 35 \\ 43 = 43 \checkmark \end{array}$$

d) $\left(\frac{1}{2}x\right)^3 - (3)^6 = (4)^6 + \left(\frac{2}{3}x\right)^6$ $CM=6$

$$\begin{array}{r} 3x - 18 = 24 + 4x \\ -4x \quad +18 \quad +18 \quad -4x \\ \hline -1x = 42 \\ \frac{-1x}{-1} = \frac{42}{-1} \\ x = -42 \end{array}$$

check:

$$\begin{array}{r} 3(-42) - 18 = 24 + 4(-42) \\ -126 - 18 = 24 - 168 \\ -144 = -144 \checkmark \end{array}$$

e) $3(2x - 1) - 2.3 = 0.04(x + 5)$

$$6x - 3 - 2.3 = 0.04x + 0.2$$

$$6x - 5.3 = 0.04x + 0.2$$

$$-0.04 \quad +5.3 \quad -0.04x \quad +5.3$$

$$\frac{5.96x}{5.96} = \frac{5.5}{5.96}$$

$$x = 0.9228$$

$$6(0.9228) - 5.3 = 0.04(0.9228) + 0.2$$

$$5.5368 - 5.3 = 0.036912 + 0.2$$

$$0.2368 \approx 0.236912$$

f) $\frac{4x-3}{3} - \frac{x-2}{6} = \frac{3+x}{2}$

$$\left(\frac{a}{b}\right)^2 - \left(\frac{b}{c}\right)^2 = \left(\frac{c}{d}\right)^2$$

$$2a - b = 3c$$

$$2(4x-3) - (x-2) = 3(3+x)$$

$$8x - 6 - x + 2 = 9 + 3x$$

$$7x - 4 = 9 + 3x$$

$$-3x \quad +4 \quad +4 \quad -3x$$

$$\frac{4x}{4} = \frac{13}{4}$$

$$x = \frac{13}{4}$$

$CM = 6$
 $a = 4x - 3$
 $b = x - 2$
 $c = 3 + x$

check: $\frac{a}{b} = \frac{4}{1} = 4$
 $7\left(\frac{13}{4}\right) - 4 = 9 + 3\left(\frac{13}{4}\right)$
 $\frac{91}{4} - \frac{16}{4} = \frac{36}{4} + \frac{39}{4}$
 $\frac{75}{4} = \frac{75}{4} \checkmark$

Example 2: Solve each problem

- a) In a jar of coins, there are 20 more nickels than quarters. The value of nickels equals the value of the quarters. How many quarters are in the jar?

- b) Nick has \$30.68 saved and earns \$11.25/week. Alice has \$24.18 saved and earns \$13.75/week. In how many weeks will they have the same amount of money?

Nick = \$30.68 + 11.25x

x = # of weeks

Alice = \$24.18 + \$13.75x

same amount of money means
when Nick = Alice

$$\begin{aligned} \$30.68 + \$11.25x &= \$24.18 + \$13.75x \\ - \$30.68 &- \$13.75x \quad - \$30.68 \quad - \$13.75x \\ - \$2.5x &= - \$6.5 \\ - \$2.5 &- \$2.5 \\ x &= 2.6 \text{ weeks} \end{aligned}$$

- c) Jack rode his bike to school at 11.2 km/hr. He returned home using the same route at 8.3 km/hr. Jack took a total of 45 min to ride to school and back. Express your answers to the nearest hundredth (to two decimal places).

I. How many minutes did Jack take to ride to school?

II. How far is it from Jack's house to school?