ENRICHING ADDITION AND SUBTRACTION FACT MASTERY THROUGH Gam

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FUSE/THINKSTOCK

Becoming fluent with basic facts is developmental. Use games infused with a focus on acquisition of strategies to help students in the early grades progress to computational fluency.

The learning of “basic facts”—single-digit combinations for addition, subtraction, multiplication, and division—has long been a focus of elementary school mathematics. Many of us remember completing endless worksheets, timed tests, and flash card drills as we attempted to “master” our basic facts as children. However, research over the past thirty years, recommendations from *Principles to Actions* (NCTM 2014) on effective mathematics teaching practices, and goals for students outlined in the Common Core State Standards for Mathematics (CCSSM) (CCSSI 2010), suggest a very different approach that has the promise of greater student engagement and success. Key to an effective approach to teaching basic facts is an understanding of the phases through which students progress as they learn their basic facts and a realization of how *meaningful* practice can be used to help students master their facts (Van de Walle, Karp, and Bay-Williams 2013). We illustrate how both of these aspects can be enacted through strategic use of games.

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Understanding the phases of learning basic facts

Fluency with basic facts can be defined as the "efficient, appropriate, and flexible application of calculation skills and is an essential aspect of mathematical proficiency" (Baroody 2006, p. 22). Baroody describes the following three phases through which students must progress as they develop mastery with a particular group of facts:

1. Modeling and/or counting all or counting on to find the answer; for example, using fingers to help keep track of their counts to solve 5 + 7 = ?
2. Deriving answers using reasoning strategies based on known facts, such as solving 5 + 7 by thinking, "Five plus five equals ten, and two more will make twelve."
3. Mastery or efficient production of answers. For example, when asked, "What is 5 + 7?" a child might call out, "Twelve," and explain, "I just knew it."

Traditional approaches to learning facts generally ignore the second phase and move children quickly from beginning conceptual experiences with addition and subtraction to rote memorization of facts via drill, flash cards, and timed testing. Although pushing students from phase 1 to phase 3 is possible, when students have not developed reasoning strategies to efficiently find a basic fact, they are unable to regenerate the answer when they forget what they have memorized. In contrast, a fluency approach allows the third phase to develop out of meaningful experiences with phase 2, as children create, share, evaluate, and practice efficient strategies for finding unknown facts from facts they have already mastered. The result is not just a much richer mathematical experience, but one necessary to establish procedural fluency (Baroody 2006; Brownell and Chazal 1935; Carpenter and Moser 1984; Henry and Brown 2008) and higher student achievement (Thornton 1978, 1990; Steinberg 1985).

CCSSM expectations for K–grade 3 use the term *fluency* in a way similar to phase 2 from Baroody's framework and as something *distinct* from automaticity. For example, in grade 1 the CCSSM expectation is as follows:

> Add and subtract within 20, *demonstrating fluency* for addition and subtraction within 10. *Use strategies* such as counting on; making ten; decomposing a number leading to a ten; using the relationship between addition and subtraction; and creating equivalent but easier or known sums. (CCSSI 2010, 1.OA.C.6)

In grade 2, children progress toward mastery. Standard 2.OA.B.2 states, "*Fluently* add and subtract within 20 using mental strategies. By end of grade 2, *know from memory* all sums of two one-digit numbers." Note the distinction between the word *fluency* and the phrase *know from memory* within the standard, which, along with the details on strategies given in 1.OA.C.6, strongly suggests that CCSSM recognizes the importance of allowing students to progress through phase 2 before expecting automaticity with their facts (phase 3).

How can teachers ensure that their students acquire the strategies needed to master phase 2? Explicitly teaching strategies does *not* mean teaching a specific strategy and then asking students to use it. Such an approach removes the reasoning from the reasoning strategy and instead adds to what a student is being asked to memorize. In fact, students in classrooms with a heavy emphasis on just memorizing basic fact strategies have been shown to have lower number sense than students whose teachers do not rely heavily on memorizing strategies (Henry and Brown 2008). Explicitly teaching strategies means *supporting* student thinking, including asking students which strategy they might use in a given situation. The key is to help students see the possibilities and then let them *choose* strategies that help them get to the solution without counting. Gravemeijer and van Galen (2003) call this approach *guided invention* because not all the reasoning strategies will be used by students without some guidance. Finally, this process takes time, often more time than we think it will take. For example, it can take between two and four lessons before most students really internalize the reasoning strategies discussed in class (Steinberg 1985). Opportunities for students to practice choosing strategies can occur in a wide variety of settings, but it is critical that practice is purposefully used. That is, a key element of developing fact fluency is "meaningful practice."

Meaningful practice

As students progress through the CCSSM expectations for K–grade 2, it is necessary for teachers to provide opportunities for meaningful practice that both engages and respects students' developmental levels. Meaningful practice of facts can come in many forms, including using story problems, ten frames, and games. Several research-based elementary curricula (for example, Everyday Mathematics 4 [Bell et al., forthcoming] and Investigations in Number, Data, and Space [Russell et al. 2008]) rely heavily on the use of games to provide engaging practice for their students, often replacing routine pencil-and-paper tasks. Children's enthusiasm for such games cannot be overestimated; very often when working with first and second graders, we have found students reluctant to stop playing when time is up! Games may be designed for either targeted practice (on a particular group of facts) or general practice (all facts for a particular operation), and the strategic use of such games can help move students along the different phases of fluency. As children play games, you can observe and interview individuals to monitor their progress through the phases (Kling and Bay-Williams 2014). These forms of assessment provide better data, while replacing the need for timed tests, which potentially have a negative impact on children (Boaler 2014). In the sections that follow, we highlight games that are motivating and useful for helping young children progress along the three phases toward meeting the CCSSM expectations for mastery of their addition and subtraction facts.

FIGURE 1

Below are brief descriptions for the High Roller game and the Roll and Total game (Bell et al., forthcoming).

(a) High Roller
Players take turns, each time rolling two regular dot dice. After the first player rolls his two dice, he determines which one has the greater number (the high roller) and keeps that die as it is while rolling the other die a second time. He then counts on from the first die to get the sum of the two dice and records the sum. The second player repeats the process, and play continues as time allows.

(b) Roll and Total
Students begin with a table with 11 columns on it, each labeled with a number 4–14. Each column has at least eight rows. (See below for a partial table.)

Working with a partner or alone, students roll two dice—one is a regular dot die, the other is a die with the numerals 3–8 recorded on it. Students then count on from the numeral die, using dots from the regular die, as needed, to find the sum, which is recorded in the appropriate column. Play continues until one column is filled completely.

	X		
			X
	X		
4	5	6	7

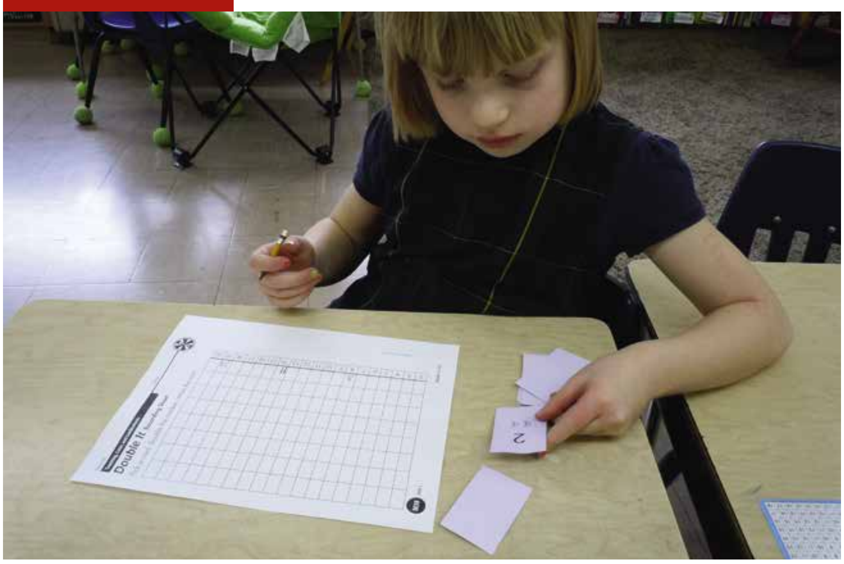
Moving within phase 1 toward phase 2

A major developmental milestone for children acquiring fluency with their basic facts is moving from counting all to counting on. This typically occurs in kindergarten or early in first grade, and it is the first strategy listed in the grade 1 standard 1.OA.C.6 (CCSSI 2010). Although it is necessary to respect the fact that this is a developmental milestone for children, and as such, cannot be "forced," we have found that two games are helpful in nudging students toward counting on. In the High Roller (see **fig. 1a**) game (Bell et al., forthcoming), children begin by rolling two dice and keeping the one with the larger number. They then roll the die with the smaller amount again and count on from the first die to find the total. Since the first die is already fixed, this game encourages students to count on as opposed to counting all when finding their sums. This skill can also be encouraged with the Roll and Total game (see **fig. 1b**) (Bell et al., forthcoming), in which children roll one die that has written numerals 3–8 and one that has the typical dots. Because the children have a number to start with, they can naturally count on using the dots from the regular die. They can then record their sums in spaces on a prepared graph (see **fig. 1c**). The counting-on strategy can be further emphasized by asking questions when students are playing the game, such as, "Why did you decide to start with the five? Why didn't you have to count those?" Thus, children who are used to counting all can be gently encouraged to adopt counting-on strategies through these games.

Developing strategies within phase 2

The major strategies used for fluently deriving addition and subtraction facts center around two main groups of facts: doubles and combinations of ten. Once students have mastered those two key groups of facts, they are able to derive every other challenging fact (see Kling [2011], for elaboration on this point). The importance of combinations of ten is echoed in CCSSM, as two of the strategies identified in 1.OA.C.6, namely "making ten" and "decomposing a number leading to a ten," require students to use their combinations of ten in fluent ways. Many textbooks "teach" such strategies, and then immediately extend to derived facts. For example, it is not uncommon for a section titled Doubles Plus One to follow the section on Doubles in a first-grade textbook. Or Combinations of Ten is followed by Making Ten. But students must *first* be fluent with the core strategies of doubles and combinations of ten before using them to derive other facts. Thus, it is important to provide students with many opportunities to practice their doubles and combinations of ten before they are expected to use them for other related strategies.

Kling found that playing Double It helped students develop automaticity with doubles facts.



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Several games have proven to be useful for such practice. First of all, for practicing combinations of ten, the Tens Go Fish game (Russell et al. 2008) is engaging for students and powerful for quickly learning combinations of ten (see **fig. 2a** for instructions). For example, in an exploratory study with eight first graders who were struggling to move beyond counting, Kling found that after three 30-minute sessions of working with ten frames and playing Tens Go Fish, the percentage of children who solved 4 + 6 by recall went from 0 percent to 50 percent. For practicing doubles, the Double It game (Russell et al. 2008) provides students with a structure within which to practice their doubles (see **figs. 2b** and **2c**). In an interview assessment with thirty-eight first graders, Kling found that doubles were the most well-known facts by the students participating in the study, all of whom had occasionally played Double It. When asked, "How did you know 9 + 9 = 18?" several students responded, "I remembered getting it in the game we played." Additional details on both games, as well as other meaningful and engaging ways to practice doubles and combinations of ten, can be found in Kling (2011). Equipped with ready recall of their doubles and combinations of ten, students such as those described above are ready to begin deriving a wealth of other addition and subtraction facts. That is, they are ready to continue progressing through phase 2 on their way to phase 3.

Moving from phase 2 to phase 3

Phase 3 is when students begin to "just know" the answer to an addition or subtraction problem. An important distinction in CCSSM is that *know from memory* does not equate to rote memorization. Rote memorization is an act of memorizing, whereas knowing from memory

FIGURE 2

Below are brief descriptions for the Tens Go Fish and Double It games (Russell et al. 2008).

(a) Tens Go Fish
This 2–4 player game is played like the Go Fish card game, but instead of looking for matching cards, children look for combinations of ten. For example, if a child has a 4 in his hand, he would ask another player, "Do you have a 6?"

Use numeral cards or playing cards, Ace–10 (Ace = 1). Children can continue to draw extra cards as needed, and play continues until all cards are used.

Children should then be encouraged to record the number sentences for some of the pairs that they found.

(b) Double It
Students begin the game by using a table (see below). The table for the Double It game has nineteen columns, each labeled with a number 2–20. Each column has eight rows.

Working alone or with a partner, students take turns drawing a card from a deck of cards containing the numbers 0–10. The student needs to double whichever number he draws and record the answer in the appropriate column on his sheet. Play continues until one column is filled completely.

2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

can result from extensive use of strategies (phase 2) until the student is so familiar with a fact, he just knows it (phase 3). In practice and in theory, mathematicians do not memorize, but come to know and understand particular facts and algorithms that are relevant to their work. Although CCSSM does not expect students to become automatic with their addition facts until the end of second grade (2.OA.B.2), students will begin operating at phase 3 with certain groups of facts much earlier than that. As discussed above, targeted practice can encourage students to become automatic with their doubles and combinations of ten within first grade. For the remaining facts, general practice can provide opportunities for students to apply their strategies frequently enough to become automatic with all facts. Making student reasoning explicit during play is a crucial aspect of making this practice mathematically meaningful. To keep the focus on strategy use during this general practice, asking questions as you observe is important:

- What reasoning strategy did you use to figure out the answer?
- Could you have used a different strategy?
- Is there a fact you know that can help you with this problem?

Such questions keep students focused on the importance of using reasoning strategies and serve as a rich source of assessment data.

The games Addition Top-It and Subtraction Top-It (Bell et al., forthcoming) serve well for general practice. These games are similar to the card game War, except that each player flips over two cards and finds the sum (or difference) of the two cards. The player with the larger sum (difference) then takes all four cards into his deck, and play continues by flipping over the next sets of cards. Once students have begun to develop some strategies for their basic addition and subtraction facts (midway through first grade), these games can begin to serve as a canvas for practicing the types of fact strategies suggested by CCSSM 1.OA.C.6. Further, children have opportunities to acquire new strategies from their fellow players if the teacher asks questions to elicit strategies used during play and follows up with prompts to the rest of the group, such as, "Do you understand how she solved it? Can you explain what she said in your own words?" Such questioning not only promotes better content understanding but also further encourages children to engage in the Standards for Mathematical Practice, particularly SMP 3: Construct viable arguments and critique the reasoning of others.

As students approach the end of first grade, the work of becoming fluent with subtraction facts can be eased by encouraging them to relate subtraction to addition. In fact, use of this relationship is explicit in CCSSM in two different standards (1.OA.C.4 and 1.OA.C.6). The game Salute! (Van de Walle, Karp, and Bay-Williams 2013) is excellent not only for mental strategy practice but also for seeing the relationship between addition and subtraction (see **fig. 3** for instructions). Notice that the players are employing the think-addition strategy to solve a subtraction fact. For example, if they heard the

FIGURE 3

When the authors play Salute! they have both players say their respective card before the round ends to ensure that every student gets to solve a problem every round.

Salute!

The objective: to be the first of the players to say what number is on your forehead (therefore winning the pair of cards).

Materials: A deck of cards numbered 0–9. We used cards from the Investigations in Number, Data, and Space series (Russell et al. 2008) because players benefit from the ten-frame illustration of the number below the numerals.

This game is for three students: a leader and two players.

1. The leader shuffles all the cards and places them facedown in a stack.
2. The leader hands one card to each player so that the player cannot see his or her own card.
3. The leader says, "Salute!" Each player places his or her card face-out on his or her forehead (players can now see the other player's card but not their own).
4. The leader says the sum of the two cards.
5. Both players determine and say what number they think is on their own forehead. The first player to correctly say the number on his forehead wins the round and places his card and the other player's card in his stack.
6. Rotate so that a new player is the leader. Repeat.

The game ends when the deck is completely used or after each player has had five turns at being the leader.

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Salute! is an excellent game for mental strategy practice as well as for understanding the relationship between addition and subtraction.

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Playing Salute! gives students opportunities to adopt reasoning that is increasingly sophisticated.

sum was 8 and saw a 5 on the other student's forehead, they would think, "Five plus what number equals eight?" When we play Salute! we ask that both players say their card before the round ends. This game provides opportunities to encourage students to adopt more sophisticated reasoning. For example, if a child is counting up for a combination for which making ten would apply (such as 8 + 5), ask ,"How else could you solve it?" Or, in a more targeted manner ask, "Can you use the making-ten strategy to find the answer?" Prompts such as these help children continue to grow in their reasoning, so that they do not become overreliant on a single, sometimes less efficient, strategy.

Differentiation

A strong message in CCSSM, as well as in NCTM Standards documents (NCTM 2000, 2006, 2014) is that *all* students must become mathematically proficient. Mastery of basic facts is foundational for becoming mathematically proficient. Yet, in any primary classroom, students will be spread across the three phases of basic fact mastery. In fact, often a single student will be at different phases for different groups of facts. Or a teacher in grades 3–5 may have a subset of students who have not reached fluency and may be seeking an engaging way to provide additional experiences for learning particular facts. Teachers must find ways to accommodate such variations through differentiation of tasks. One way to differentiate is by the selection of games appropriate for where the student is developmentally (as described above). Another way is through modification of games, as slight shifts in the game rules or the cards used can affect the level of difficulty (Forbringer and Fahsl 2010). We have found that Addition Top-It, Subtraction Top-It, and Salute! can all be easily modified in a variety of ways. The following are suggestions to differentiate these games (and others):

- Use visual tools that provide a concrete model, such as a ten frame, double ten frame, or number line.
- Alter decks of cards, using only smaller numbers or building to larger numbers.
- Add in wild cards (a first grader's idea) so that students can adapt the level of difficulty themselves.
- Require explicit sharing of how students found sums, differences, or missing addends.
- Ask students to monitor which strategies they are using and how often (they can keep a tally).
- Extend beyond basic facts, such as using three addends in Addition Top-It.

Notice that several of these modifications make strategy use explicit and focus on student self-monitoring, and all provide ways to help keep practice meaningful and engaging. This helps students realize which strategies and which facts are their strengths and which ones they must continue to develop, promoting their continued growth toward developing fact mastery. Finally, these modifications can be applied to many classic games, such as Bingo, Concentration, Dominoes, Four-in-a-Row, and Old Maid (see Van de Walle, Karp, and Bay-Williams [2013] for details on differentiating basic fact instruction with these games).

These students are playing Addition Top It, a good game for general practice.

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Communicating with families

Many adults with childhood experiences learning mathematics through rote memorization are unaware of the value of developing reasoning strategies to support fact fluency (phase 2). Therefore, it is critical that teachers share with parents this important process for learning facts as well as the role that games can play in the process. The games shared in this article are great for Family Math Nights, homework, summer practice, and for parent volunteers to use as they work with students in your class. It is important to communicate to parents the expectation of having students use and explain the various strategies that are being developed in class. Also, the Families Ask article "Rules or Understanding?" Martinie (2005) addresses the parent question, "Isn't it more efficient for children to learn rules and have understanding come later when they are more mature?" Although published in the *Mathematics Teaching in the Middle School* journal, this one-page, parent-friendly handout is appropriate for all levels. (See the more4U box on **p. 247**.) Sharing resources such as this with families can illuminate the importance of developing conceptual understanding as a part of mathematical proficiency.

Understanding the progression

Given the focus within CCSSM on mathematical proficiency for all students, understanding the phases that students progress through when learning basic facts is critical, as is how to help students navigate this progression in meaningful ways. Games can help students develop and practice new approaches as they transition from using concrete tools to counting to using mental reasoning strategies. But games support strategy development only when the use of reasoning strategies is explicitly built into the games and reinforced through student-teacher and student-student interactions. When this happens, students' reasoning strategies develop along with their motivation, interest, and desire to be challenged.

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