Measurement of Length: How Can We Teach It Better?

Measurement of length is taught repeatedly starting in kindergarten and continuing in grades 1, 2, and beyond. However, during the past twenty-five years, according to the National Assessment of Educational Progress (NAEP), the outcome of this instruction has been disappointing. As can be seen in **table 1**, only 14 percent of the third graders and half (49 percent) of the seventh graders gave the correct answer—5 cm—to a question on the 1985–1986 NAEP (Lindquist and Kouba 1989). Similar items included in other NAEPs before and after this one have produced similar findings.

What is so hard about measurement of length? **Table 1** is informative because it reveals the incorrect answers the students gave. Thirty-seven percent of the seventh graders gave the answer 6 cm, evidently determined by counting the numerals 3, 4, 5, 6, 7, and 8. This answer indicates that more than a third of the seventh graders did not know what a unit of length was, especially the segment measuring 1 cm. About a third (31 percent) of the third graders made the same error, and another third (30 percent) chose the answer 8 cm, obviously paying attention only to the end of the line.

The purpose of this article is to explain, on the basis of research, why instruction has been ineffective and to suggest a better approach to teaching. Following the model of Piaget, Inhelder, and Szeminska (1960), I individually interviewed and videotaped 383 children in grades 1–5 in two public schools in a low-to-middle-income neighborhood in the South (Kamii and Clark 1997). Each child was given a sheet of paper (11 by 17 inches) with an inverted T photocopied on it (see **fig. 1**). Although the vertical line appeared longer—the result of a perceptual illusion—both lines were 8 inches long. This task was designed to find out how the child went about comparing two lines that could not be compared directly. Because the lines could not be moved and placed side by side, the child had to use an object such as a ruler or a strip of paper to make an indirect comparison.

The interview procedure consisted of the following four steps:

1. *Perceptual judgment.* Presenting the child with the figure of the inverted T (**fig. 1**), the interviewer asked, "Do you think this line (line *A*) is as long as this line (line *B*), or is this one (*A*) longer, or is this one (*B*) longer?" The purpose of these questions was to spur the child's involvement in the task and give him or her reasons for answering the subsequent questions.

2. *Transitivity* (first attempt). With a tagboard strip (12 by 0.5 inches) in hand, the interviewer asked, "Can you use this to prove (or show) that this line (*A*) is longer than the other line [or whatever the child had just said]?" This question was asked to find out if the child could demonstrate transitive reasoning by using the strip. Transitive reasoning refers to the ability to reason logically that if *A* is equal to the length indicated on the strip and the length indicated on the strip is equal to *B*, then *A* and *B* can be inferred to be equal. Piaget et al. (1960) had shown that children are not able to make this logical inference before the age of seven or eight; younger children said that the only way to compare the two lengths was to put them side by side for direct comparison.

3. *Unit iteration.* Offering a small block (1.75 by 0.88 by 0.25 inches) to the child, the interviewer

By Constance Kamii

Constance Kamii, ckamii@uab.edu, is professor of early childhood education at the University of Alabama at Birmingham. She studied under Jean Piaget off and on for fifteen years and has been using his theory to improve mathematics education in kindergarten and the early grades.

asked, "Can you use this to prove (or show) that this line (*A*) is as long as the other one [or whatever the child had just said]?" The purpose of this question was to find out if the child was able to compare the two lengths by using the small block as a unit to iterate. (The child had to make a mark on the paper to indicate the top of the block and then move the block up without any gap or overlap between the units. The 8-inch lines were about 4.5 blocks long.) The strip was used to find out if the child could compare whole lengths, but the block was used to determine if the child could think about the 1.75-inch length as a part to iterate within the whole length.

4. *Transitivity* (second attempt). This question was posed only to those children who were unsuccessful with the strip used in the second step. Four additional blocks of the same size were given to the child, and he or she was asked, "Can you use these to prove (or show) that this line (*A*) is as long as the other one [or whatever the child had just said]?" This question was asked to find out if the child could demonstrate transitive reasoning when offered a second chance.

Table 2 shows the findings from these interviews. The percentages of students who demonstrated transitive reasoning can be seen in the third, fourth, and fifth columns, labeled "Transitive reasoning." The third column (titled "With strip") presents the percentages who demonstrated transitive reasoning with the strip, and the fourth column (titled "With blocks") shows those who demonstrated it with five blocks (but not with the strip). The total percentages who demonstrated transitive reasoning can be seen in the fifth column, labeled "With strip or blocks." The percentages in this column indicate that most children (72 percent) construct transitive reasoning by second grade. These findings support the age reported by Piaget et al. (1960).

The last column of **table 2** shows that unit iteration develops gradually and that most children (76 percent) construct it by fourth grade. Each interview was reviewed to find out if anyone demonstrated unit iteration before transitive reasoning, and no such case was found. Our research thus supported Piaget's statement that children construct unit iteration out of transitive reasoning.

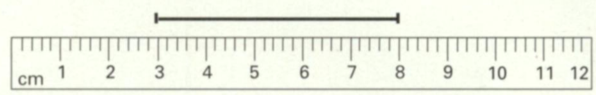
Why Has Instruction Been So Ineffective?

Children's development of logic can be used to explain some of the poor results reported by NAEP.

Table 1

Percentages of Students in Grades 3 and 7 Responding to an Item of the National Assessment of Educational Progress

Table 5.3
Rulers



Item	Percent Responding[a] Grade 3	Grade 7
How long is this line segment?[b]		
3 cm	4	1
5 cm *	14	49
6 cm	31	37
8 cm	30	9
11 cm	6	2
I don't know.	15	2

[a] The response rate was .80 for grade 3 and .97 for grade 7.
[b] An actual centimeter ruler was pictured.
* Indicates correct response.

Table 2

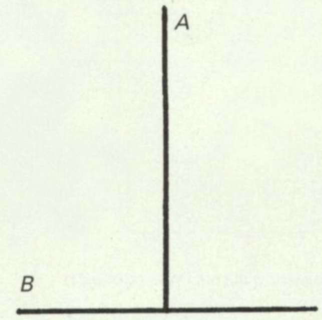
Percentages of Students Who Demonstrated Transitive Reasoning and Unit Iteration

Grade	*n*	Transitive reasoning			Unit iteration
		With strip	With blocks	With strip or blocks	
1	78	21	8	29	10
2	79	56	16	72	33
3	75	77	8	85	55
4	75	83	1	84	76
5	76	89	3	92	78

Because unit iteration was not demonstrated by the majority of students before fourth grade (see **table 2**), it is not surprising that, as shown in **table 1**, only 14 percent of the third graders got the correct answer—5 cm—because it involved units. However, the answer 8 cm, which 30 percent of the third graders gave, cannot be explained by the data in **table 2**. Because 72 percent of the second graders demonstrated transitive reasoning, most of the third graders should have been able to relate the whole length of the line to the corresponding part of the ruler. The low-level response given by so many third graders must be attributed to ineffective teaching. In seventh grade, the major problem was that 37 percent did not understand units of length and gave the answer 6 cm.

Figure 1

An inverted T used in the indirect comparison task



Two aspects of instruction explain why so many seventh graders did not understand units of length and why so many third graders paid attention only to one end of the line. First, teachers almost always ask students to produce a number about single objects rather than asking for a comparison of two (or more) objects. For example, young children are often asked to line up paper clips from one end of a pencil to the other end and then asked, "How many paper clips long is the pencil?" In such a situation, because there is no need to measure the pencil, children can feel the need to produce a number only because the teacher wants a number. Second, instruction teaches empirical procedures without logical reasoning. Lining up paper clips and counting them is an empirical procedure requiring no logical thinking. Asking children to move a yardstick across the chalkboard is likewise the teaching of a mere empirical procedure. Each of these shortcomings of instruction is elaborated here.

Comparing Two or More Objects Indirectly

Measurement is unnecessary when we are dealing with only one object or when two objects can be compared directly. To compare the length of two pencils directly, for example, all we have to do is hold them side by side. Measurement becomes necessary when we want to compare two or more objects indirectly. This is why the following two kinds of activities are better than those typically found in textbooks—for example, questions such as "How wide is your desk?" and "How long is the chalkboard?"

1. *Asking for indirect comparisons for a purpose.* Children can be asked exactly how much paper they need to bring from another room to cover the bulletin board. In this situation, children have to think of a tool to use to make an indirect comparison. Students are likely to think about a piece of string, a paper strip, a stick, a book, a yardstick, or a 12-inch ruler. Debate about the quantity of paper to bring, without wasting any, will motivate students to think logically about the length and width of the paper they need to bring.

When students are asked if a doorway is wide enough for a certain table to go through or if a certain space is big enough to put a large carton in, they also have to make an indirect comparison. The problems in **figure 2** also require indirect comparisons, and students can be asked to make drawings that are twice the size of a photocopied model to take home and amuse their families.

One day one teacher commented to another that the fourth graders had larger desks than the third graders, but the second teacher disagreed. This was an excellent opportunity for students to make indirect comparisons for a purpose. The students began by disagreeing about the size of the desks in their own classroom! Debating to convince one another is an excellent way to develop children's logic. Their logic develops when they are encouraged to think hard.

2. *Measuring out.* Measuring is what we do when we do not know the exact length of an object. Measuring out is what we do when we know the number of units but cannot imagine what 100 feet looks or feels like, for example. Children may be encouraged to produce various lengths when they read that a whale or dinosaur was so many feet long or that the *Mayflower* was so many feet long. This activity involves only one object but is useful because children want to know how large the object is. We must be aware that transitive reasoning is only implicitly involved in these activities. If children are not bothered by gaps or overlaps between the "units" they measure out, this behavior is evidence that they are not thinking about them as parts of a whole length. Correcting their behavior in such a situation is not helpful if the child does not feel the logical necessity of iterating the unit without any gaps or overlaps. (Unit iteration generally occurs around fourth grade.)

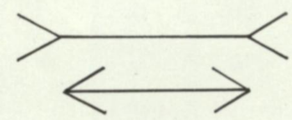
Children like to make things, and measuring out is necessary when the teacher suggests projects such as making spring flower baskets with paper, beanbags with cloth, or kites with dowel rods, cloth, string, and a small metal washer. Many books can be found with "recipes" detailing simple or complex measurement involving whole numbers or fractions and decimals. These activities are good because children are motivated to make things, and they often notice their own errors when their measurement is inaccurate.

Making a kite involves measuring out, but many other science activities can involve measuring. For example, in studying evaporation, children can be asked to measure the size of a container and the depth of the water in it to answer questions such as these: Does evaporation depend on the size of the container? Does it depend on the size of the surface area? How do you measure evaporation anyway? Many other science activities should be reviewed not only from the standpoint of measurement but also from the viewpoint of children's development of logic and motivation.

It was pointed out earlier that the third graders' answer—8 cm—and the seventh graders' answer—6 cm—shown in **table 1** are results of ineffective instruction. In comparing two objects indirectly, children will have to think about the whole length of each object or the units within each length. In measuring out for a purpose, too, they will have to think about the whole length in question and the units within each whole.

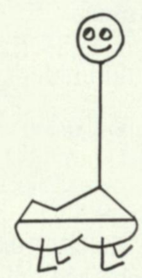
Figure 2

Equal lengths that produce the illusion of being unequal



Which horizontal line is longer?

Or are they the same in length?



Is the body (horizontal line) shorter than the neck?

Empirical Procedures vs. Reasoning

Teachers' guides often say that if children do not align the edge of a ruler with the edge of the object to be measured, the teacher should tell them to align the two edges. This is an example of teaching an empirical procedure to correct a surface behavior. If children do not align the two edges on their own, this is evidence of the absence of transitive reasoning. If 30 percent of the third graders looked only at the end of the line to be measured, this may well be the result of having learned only empirical procedures.

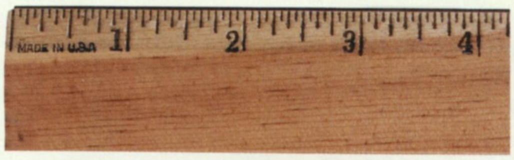
As stated earlier, 37 percent of seventh graders (see **table 1**) did not know what a unit of length was, particularly the unit between 0 and 1. This is a truly shocking finding in view of the fact that 76 percent of fourth graders (see **table 2**) could reason about units. The measuring-out activities described earlier are likely to make children think about units within a whole length.

To clarify what empiricist teaching is and why it is undesirable, it is necessary to review the fundamental distinction Piaget made among three kinds of knowledge according to their ultimate sources: physical knowledge, logico-mathematical knowledge, and social-conventional knowledge. Physical knowledge is knowledge of objects in the external world. Our knowledge of the weight and the color of a pencil is an example of physical knowledge. The fact that a pencil is made of wood while a paper clip is made of metal is also an example of physical knowledge. Examples of social-conventional knowledge are our knowing that Americans use inches while Canadians use centimeters and that a pencil is inappropriate for a signature in a contract. Social knowledge has its source in conventions made by people, but physical knowledge has its source in objects.

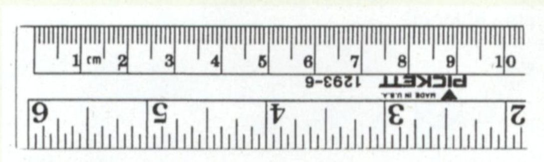
Logico-mathematical knowledge consists of mental relationships and originates in each individual's mind. For example, if we are presented with two unsharpened pencils, one yellow and one white, we can say that they are different. "Different" is a mental relationship that is not observable with our eyes. The pencils are observable (physical knowledge), but the difference between them (logico-mathematical knowledge) is not. The proof is that if we decide to ignore color, we can say that the two pencils are similar or that they are the same in length or weight. A fifth mental relationship we can make is about the numerical relationship "two"

Figure 3

Two types of rulers



a. Common ruler where the 0 (starting point) is implicit at the edge



b. Engineering or scientific ruler where the 0 (starting point) is implicit and indented

itself. "Different," "similar," "the same in length," "the same in weight," and "two" are all mental relationships that have their source inside each individual's head. Logico-mathematical knowledge thus consists of mental relationships, which are not empirically observable. It is possible to see empirically that paper clips have been lined up end to end, but it is not possible to see only units with our eyes.

With respect to units, teachers' guides often say that nonconventional units should be taught before conventional units. However, once a child has the logico-mathematical knowledge of units, conventional units can be used as easily as unconventional units. If centimeters are hard to teach, unconventional units are equally hard for children who do not have the logico-mathematical knowledge of units.

A few words about the 12-inch rulers that most children use in school: These rulers look like the one in **figure 3a**. Because the 0 is only implicit on this ruler, children do not have to think about the unit between 0 and 1, especially if they are told to align the edge of the ruler with the edge of the object to be measured. This problem may in part explain the answer 6 cm that 37 percent of the seventh graders gave in **table 1**. **Figure 3b** shows the kind of ruler that engineers use, which has an implicit 0 away from the edge. Because this kind of ruler is more expensive, teachers may want to white out all the numbers on the inexpensive rulers so that students will have to think about units. Another possibility is to ask students to make their own rulers and yardsticks. If children make their own rulers and use the inches and centimeters marked on these to make toys, such as a race car made from a milk container, they will be motivated to think about units more thoughtfully than when they are asked how long a pencil is.

Principles and Standards for School Mathematics (National Council of Teachers of Mathematics 2000) defines measurement as "the assignment of a numerical value to an attribute of an object, such as the length of a pencil" (p. 44). When *Principles and Standards* later presents an anecdote, the outcome of this conception of measurement becomes evident. A teacher had given her class a list of things to measure. One of the students, Mari, had a pencil that was obviously shorter than her book but wrote that both objects were 12 inches long. When the teacher tactfully commented on these numbers, Mari replied, "You're right.... The book is longer, but they are both twelve inches" (p. 106). If Mari had been asked to compare two objects that could not be compared directly, she would not have made such a nonsensical statement.

Measurement was invented by our ancestors for the purpose of making indirect comparisons. There is a parallel between humankind's construction of measurement and each individual's construction of measurement. Lengths can be compared either by comparing whole lengths (with transitive reasoning) or by comparing the number of units within each whole. Piaget et al. (1960) showed that unit iteration grows out of transitive reasoning. The time has come for educators to rethink the notion that "measurement is the assignment of a numerical value to an attribute of an object, such as the length of a pencil" (NCTM 2000, p. 44). If we improve the way we teach measurement, results of future National Assessments of Educational Progress should also improve.

References

Kamii, Constance, and Faye B. Clark. "Measurement of Length: The Need for a Better Approach to Teaching." *School Science and Mathematics* 97 (March 1997): 116–21.

Lindquist, Mary M., and Vicky L. Kouba. "Measurement." In *Results from the Fourth Mathematics Assessment of the National Assessment of Educational Progress*, edited by Mary M. Lindquist, pp. 35–43. Reston, VA: National Council of Teachers of Mathematics, 1989.

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics.* Reston, VA: NCTM, 2000.

Piaget, Jean, Barbel Inhelder, and Alina Szeminska. *The Child's Conception of Geometry.* 1948. Reprint, London: Routledge and Kegan Paul, 1960. ▲