Template - Lesson Plan – Backwards by Design

Subject/Grade: Foundations & Pre- Calc 10 **Lesson Title:** #3: Measuring an Inaccessible Height **Teacher(s):** Miss Doratti

Stage 1: Identify Desired Results

Outcome(s)/Indicator(s): FP10.4

a) Students can develop, explain, and apply relationships and how side and angle sizes compare in similar right triangles
b) Students can show how to find the hypothenuse (longest side) of a right triangle and the legs (adj and opp sides) in a right triangle
c) Students can solve problems using one or more right triangles by applying ratios or the Pythagorean theorem
d) Students can create and solve problems that involve primary trigonometric ratios, Pythagorean theorem, and other measurements

Key Understandings: ('I Can' statements)

- I can develop, explain, and apply relationships between the ratios of side lengths and angle sizes in a similar right triangle.
- I can show how to find the hypotenuse of a right triangle.
- I can solve problems using one right triangle.
- I can create and solve problems with right triangle

Key Questions:

When we solve a right triangle, sometimes we determine the measure of an unknown angle before we determine the length of an unknown side and sometimes, we reverse these calculations. How would you decide which measure to calculate first?

Can we solve a right triangle if we are given only the measures of the two acute angles? Explain.

What is the advantage of determining the unknown angle before the unknown sides?

Prerequisite Learning:

- SOHCAHTOA – need to understand what this means and how it relates to ratios.
- All angles in a triangle add up to 180 degrees.
- Basic mathematics to remove denominators when solving for a variable and how it affects the other numbers in an equation.
- What the tangent ratio is and how to calculate lengths using it.
- What cosine and sine ratios are and how to calculate lengths using it.

Stage 2: Determine Evidence for Assessing Learning

Formative:

- Daily Problem/Group Work: Groups will have 3 people per group (1 writing on the board, 1 giving instructions to the writer, and one using pen and paper to solve it.) and they will have to work together to solve. I will prompt them to switch, and they can use this method to fix others mistakes until they come to a solution. They will be assessed based on their understanding of the knowledge. I will have a lot of time to do this assessment because I would be assessing them individually as they participated. I will use a checklist to record checkmarks and notes on each students' abilities. *This day the two groups of students that created a problem, will use their problems to run this activity. They will be assessed separately because they will hand in the problem they created and the solution. They must be sent to me via Google Classroom (as a photo) the night before so that I can check its credibility. There will be peer, self, and teacher assessment for the daily problem.

Summative:

- Exit Slip: Students will have a one question exit slip. This will be used as a checkpoint for that daily teaching. I will be able to re-teach and/or fill gaps in knowledge prior to moving on to the next lesson. This will also make sure that no one gets left behind as I focus on moving forward. We will also go through the solution for the exit slip first thing the next day. I will use a checklist with students' names to record a checkmark and notes.

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Stage 3: Build Learning Plan

Set (Engagement): **Length of Time:** 5-10 min

Presentations from groups for daily problem (2)

Group activity:

5 Groups of three will be given a triangle with missing lengths or angles. They will do this up on the board. One person will be the board writer, one will be the verbal instruction giver, and the other will be at a desk with paper and pencil working on the solution. Teacher will say switch and they must switch roles. The writer cannot give solutions or help solve the problem at all. As they switch, they may revise the previous work. Go until they believe they have the solution. Groups will stay quiet until all groups have come up with a solution. Repeat with the other 5 groups of 3. Go through the solution that is correct. Discussion after each round is necessary to use this as assessment as learning.

Development: **Length of Time:** 10-20 min

Definitions:

Solving a triangle: to determine the measures of all of the angles and the lengths of the sides in the triangle

Group Activity:

THINK ABOUT IT

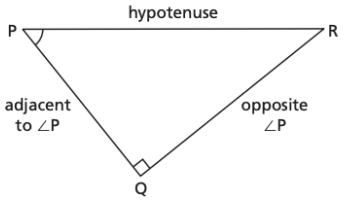
Work with a partner.

Anwar is designing a wheelchair accessibility ramp for his sister. He knows these data:

- The ramp will rise 1 ft. from the level ground to the door of the house.
- The horizontal distance from the start of the ramp at the sidewalk to the door is 20 ft.
- The building code states that the angle of inclination of the ramp must be less than 5°.

Determine whether Anwar's design will comply with the building code.

Discussion to follow



$$\tan P = \frac{\text{opposite}}{\text{adjacent}}$$

$$\sin P = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos P = \frac{\text{adjacent}}{\text{hypotenuse}}$$

Instructional Strategies:

- Group work activity
- Group discussion
- Google Classroom exit slip
- Guided examples
- Pairs – daily problem (2 groups)
- Student-created example via some sort of media (different per student) – homework

Materials/Resources:

Google Classroom
White Board
Textbook
Computers/Devices for online work

Possible Adaptations/ Differentiation:

Universal design will be used so there should not be any adaptations, students will be able to answer using their current knowledge.

Management Strategies:

These will have been in place prior to this lesson. They are working on where to hand things into and where to find missing notes/assessments. Where to put phones for class. Where to find supplies in the classroom.

Safety Considerations:

Responsible use of whiteboard markers. Appropriate language use in the group.

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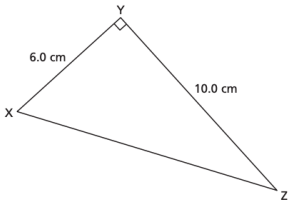
Guided example:

Example 1 Solving a Right Triangle Given Two Sides

Solve $\triangle XYZ$. Give the measures to the nearest tenth.



SOLUTIONS

Method 1

Determine the length of XZ first.
Use the Pythagorean Theorem in right $\triangle XYZ$.

$XZ^2 = 6.0^2 + 10.0^2$

$XZ^2 = 36.00 + 100.00$

$XZ^2 = 136.00$

$XZ = \sqrt{136}$

$XZ = 11.6619\ldots$

XZ is approximately 11.7 cm.

Determine the measure of $\angle Z$.

Since YZ is adjacent to $\angle Z$ and XZ is the hypotenuse, use the cosine ratio.

$$\cos Z = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos Z = \frac{YZ}{XZ}$$

$$\cos Z = \frac{10.0}{\sqrt{136}}$$

$\angle Z = 30.9637\ldots°$

So, $\angle X = 90° - \angle Z$

The acute angles in a right triangle have a sum of 90°.

$\angle X = 59.0362\ldots°$

Method 2

Determine the angle measures first.
Determine the measure of $\angle Z$ in right $\triangle XYZ$.

Since YZ is adjacent to $\angle Z$ and XY is opposite $\angle Z$, use the tangent ratio.

$$\tan Z = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan Z = \frac{XY}{YZ}$$

$$\tan Z = \frac{6.0}{10.0}$$

$\tan Z = 0.6$

$\angle Z = 30.9637\ldots°$

So, $\angle X = 90° - \angle Z$

The acute angles in a right triangle have a sum of 90°.

$\angle X = 59.0362\ldots°$

Determine the length of XZ.

Since YZ is adjacent to $\angle Z$ and XZ is the hypotenuse, use the cosine ratio.

$$\cos Z = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos Z = \frac{YZ}{XZ}$$

$\cos 30.9637\ldots° = \frac{10.0}{XZ}$ Solve the equation for XZ. Multiply both sides by XZ.

$XZ \cos 30.9637\ldots° = 10.0$ Divide both sides by $\cos 30.9637\ldots°$

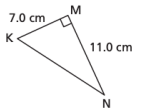
$$XZ = \frac{10.0}{\cos 30.9637\ldots°}$$

$XZ = 11.6614\ldots$

XZ is approximately 11.7 cm.
$\angle X$ is approximately 59.0° and
$\angle Z$ is approximately 31.0°.

CHECK YOUR UNDERSTANDING

1. Solve this triangle. Give the measures to the nearest tenth.

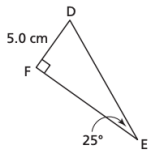


[Answers: KN ≐ 13.0 cm; $\angle K \doteq 57.5°$; $\angle N \doteq 32.5°$]

Which other trigonometric ratio could you have used in Method 1? Why might it be better to use this ratio?

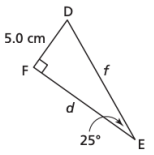
Example 2 Solving a Right Triangle Given One Side and One Acute Angle

Solve this triangle. Give the measures to the nearest tenth where necessary.



SOLUTION

Label a diagram.
Determine the measure of ∠D first.
In right △DEF:



$\angle D + \angle E = 90°$

$\angle D = 90° - 25°$

$\angle D = 65°$

Determine the length of EF. Since EF is opposite ∠D and DF is adjacent to ∠D, use the tangent ratio.

$$\tan D = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan D = \frac{d}{e}$$

$$\tan 65° = \frac{d}{5.0}$$ Solve the equation for d. Multiply both sides by 5.0.

$$5.0 \tan 65° = d$$

$$d = 10.7225\ldots$$

EF is approximately 10.7 cm.

Use the sine ratio to calculate the length of DE.

$$\sin E = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin E = \frac{e}{f}$$

$$\sin 25° = \frac{5.0}{f}$$ Solve the equation for f. Multiply both sides by f.

$$f \sin 25° = 5.0$$ Divide both sides by sin 25°.

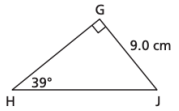
$$f = \frac{5.0}{\sin 25°}$$

$$f = 11.8310\ldots$$

DE is approximately 11.8 cm.

CHECK YOUR UNDERSTANDING

2. Solve this triangle. Give the measures to the nearest tenth where necessary.

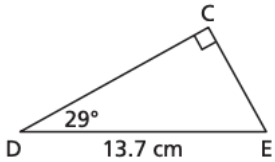


[Answers: ∠J = 51°; GH ≐ 11.1 cm; HJ ≐ 14.3 cm]

What is the advantage of determining the unknown angle before the unknown sides?

Learning Closure: **Length of Time:**

Exit slip: This will be posted on the board via Google Classroom.
Solve the right triangle. Give the measures to the nearest tenth (one decimal place). Reminder: this means finding all the angles and the side lengths.



Hand this into your bin. Please ensure your name is on it.

Stage 4: Reflection

(This part of the lesson is completed after the lesson has been delivered; this is where you can record how it went, what you would keep, and what would you change for next time)